Broadband

Data Sheet

Telewest Communications is a leading broadband cable communications operator that provides multi-channel television, telephone and Internet services to millions of UK houses. It also provides voice and data telecommunications services to over 60 000 business customers.

In March 2000 Telewest Communications launched Blueyonder, the UK's first high speed broadband Internet service for home personal computers. Since then the number of broadband connections in the UK has grown rapidly.

The table below shows this growth over a 3 year period from January 2001 to January 2004.

y represents an estimate of the total number of broadband connections in thousands and x represents the number of 6 month time intervals since January 2001.

Time (6 mth intervals) x = 0 on 1st January 2001	No. of Connections (thousands)
x	y
0	110
1	200
2	370
3	830
4	1500
5	2370
6	3200

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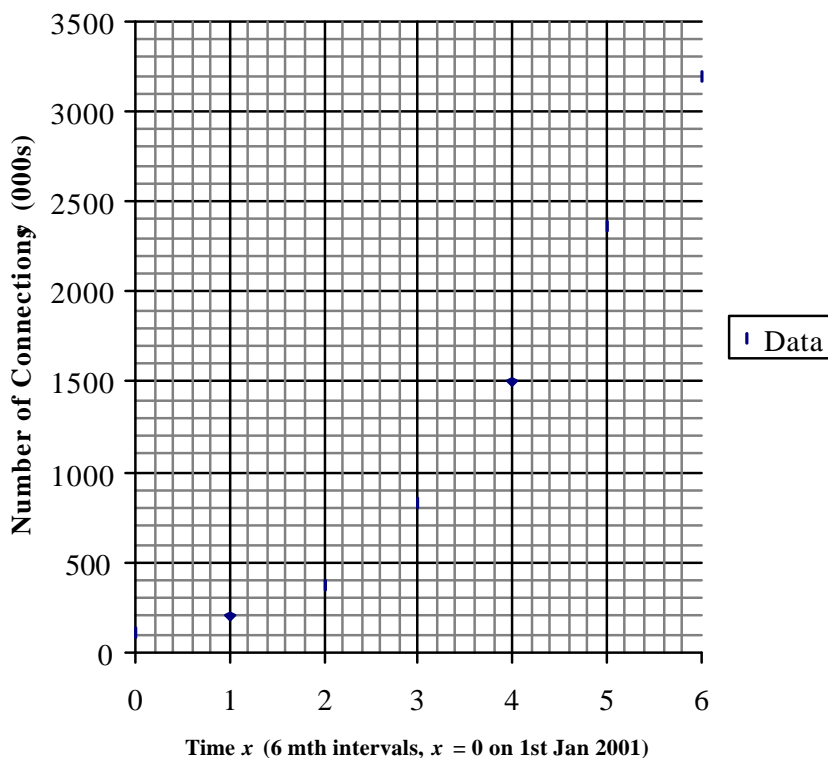
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Broadband

To find a quadratic model using algebra

Growth in UK Broadband Connections

The graph shows the data given on the Data Sheet.

There are a number of different types of function that could be used to model this data.

This activity shows how to use algebra to find a **quadratic** model i.e a model of the form $y = ax^2 + bx + c$.

(graph: Number of Connections (000s) vs Time x (6 mth intervals, x = 0 on 1st Jan 2001), Data)

Choose 3 data points, say (0, 110), (3, 830) and (6, 3200)

Substitute these values into the quadratic equation $y = ax^2 + bx + c$

Show that this gives: $c = 110$ and the simultaneous equations: $3a + b = 240$
$$6a + b = 515$$

Solve the simultaneous equations to find a and b.

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How good is the quadratic model?

You should find that the algebraic method outlined above gives the quadratic model

$$y = 91\tfrac{2}{3}x^2 - 35x + 110 \quad \text{(or } y = 91.\dot{6}x^2 - 35x + 110\text{)}$$

To check how good this model is, first find the values it predicts for the values of x in the original data.

For example, when $x = 2$ (i.e. on 1st January 2002)

$$y = 91\tfrac{2}{3} \times 2^2 - 35 \times 2 + 110$$

$$= 406\tfrac{2}{3} = 407 \text{ (to 3sf)}$$

x	y from original data	y from model	% Error
0	110		
1	200		
2	370	407 (to 3sf)	10%
3	830		
4	1500		
5	2370		
6	3200		

This has been entered into the table above alongside the original data.
Complete the column to show the values of y given by the model for the other values of x.

Use these values to draw the curve $y = 91\tfrac{2}{3}x^2 - 35x + 110$ *on the graph on the previous page. Compare the model with the original data values.*

You can also describe how close a model is to the actual data using percentage errors.

$$\boxed{\% \, \text{error} = \frac{\text{predicted value} - \text{actual value}}{\text{actual value}} \times 100}$$

For example, when $x = 2$, the model estimates the number of broadband connections to be $406\tfrac{2}{3}$ thousand whereas the initial data gives 370 thousand.

$$\% \text{ error} = \frac{406\tfrac{2}{3} - 370}{370} \times 100 = 10\% \text{ (nearest \%)}$$

Note the fact that the % error is positive means that the model's prediction is 10% too high, whereas a negative % error would mean the prediction was too low.

Calculate the % errors for other values of x and write them in the table above.

It is advisable to check how well a model is likely to predict values in the future.
Calculate the number of broadband connections the model predicts for x = 10 and x = 20
Do you think the values you have found are realistic?

You can also check how well the model estimates the number of connections before 2001 by using negative values of x.
Use the model to calculate y when x = – 1.
Explain what information this gives and say whether or not you think it seems realistic.

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Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes
The data on the Data Sheet shows how the number of broadband connections has grown in recent years. It is the same as that given in Broadband A and B, but this activity shows how to use algebra to find a quadratic model, whereas Broadband A uses Excel and Broadband B uses a graphic calculator. You can use just one of the activities, two of them or all three. If you use A or B as well as this activity, you could discuss with students how Excel and the graphic calculator give a better model by using all of the points and finding the best quadratic curve by regression.

Answers $a = 91\frac{2}{3}$, $b = -35$ and $c = 110$

It should be no surprise that the values given by the model agree with the data at $x = 0$, 3 and 6, but not at the other values. Most of the other values given by the model are below the actual data values.

x	y from original data	y from model	% Error
0	110	110	**0%**
1	200	167 (to 3sf)	**− 17%**
2	370	407 (to 3sf)	**10%**
3	830	830	**0%**
4	1500	1437 (to 3sf)	**− 4%**
5	2370	2227 (to 3sf)	**− 6%**
6	3200	3200	**0%**

The graph shows the quadratic function

$y = 91\frac{2}{3}x^2 - 35x + 110$

is a good model of the data.

When $x = 10$, the model predicts $y = 8\,930$ (to 3 sf)
When $x = 20$, the model predicts $y = 36\,100$ (to 3 sf)
These values, being **in thousands** seem unrealistically high, bearing in mind that the population is less than 60 million.

When $x = -1$, the model gives $y = 237$ (to 3 sf).
This is not consistent with there being only 110 thousand connections 6 months later.

Growth in UK Broadband Connections

Model: $y = 91\frac{2}{3}x^2 - 35x + 110$